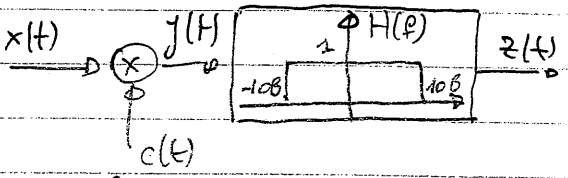
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$x(t) = e^{-t/\tau}\cos(2\pi f_0 t)\,u(t)$ | $f_0 = 4B$

$c(t) = T\sum_n \delta(t-nT)$ | $T = 1/4f_0$

$B = 1/2\pi\tau$

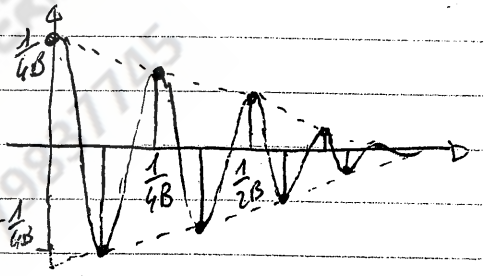
Calcolare $X(f)$, $z(t)$.

$x(t) = x_0(t)\cos(2\pi f_0 t) \;/\; x_0 = e^{-t/\tau}u(t) \rightarrow X_0(f) = \frac{\tau}{1+2j\pi f\tau}$

Applico il teorema modulazione: $X(f) = \frac{\tau}{2}\left[\frac{1}{1+j2\pi(f-f_0)\tau} + \frac{1}{1+j2\pi(f+f_0)\tau}\right]$

$c(t) = \frac{1}{16B}\sum_n \delta\left(t-\frac{n}{16B}\right) \longrightarrow y(t) = \frac{1}{16B}\sum_n x\left(\frac{n}{16B}\right)\delta\left(t-n/16B\right)$

Disegno i 2 segnali $y(t)$, $x(t)$ - $y(t)$ è campionato da una serie di delta d'intervallo $1/2B$, per cui posso esprimere $y(t)$ come:



$$y(t) = \frac{1}{16B}\underbrace{e^{-t/\tau}u(t)}_{x_0(t)}\left[\sum_n \delta\left(t-\frac{n}{4B}\right) - \sum_n \delta\left(t-\frac{n}{4B}-\frac{1}{8B}\right)\right]$$

$$Y(f) = \frac{X_0(f)}{4}\otimes\sum_k \delta(f-4Bk) - \frac{X_0(f)}{4}\otimes\sum_k \delta(f-4Bk)e^{-j\pi k} =$$

$$= \frac{1}{4}\sum_k X_0(f-4Bk)\left(1-(-1)^k\right) = \begin{cases} 0 & k \text{ pari} \\ \frac{1}{2}X_0(f-4Bk) & k \text{ dispari} \end{cases}$$

Il filtro $H(f)$ fa passare solo le componenti con $k = \pm 1$ visto che per $k = \pm 2$ $Y(f) = 0$ e per $k = \pm 3$ avrei $\pm 12B$ che risulta fuori dal filtro

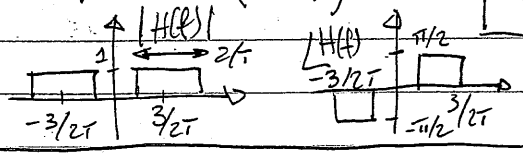
$$Z(f) = \frac{1}{2}X_0(f-4B) + \frac{1}{2}X_0(f+4B).$$

Antitrasformo: $z(t) = x_0(t)\cos(2\pi f_0 t) \rightarrow z(t) = \Re(t)$

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$x(t) = \sum_n x_0(t - nT)$ $\quad x_0(t) = 3\,\mathrm{sinc}^2(Bt) + \frac{1}{4}\delta\left(t - \frac{T}{4}\right)$ $\quad B \triangleq 3/T$



Calcolare $z(t)$, P_z

$$X_0(f) = \frac{3}{B}\left(1 - \frac{|f|}{B}\right)\mathrm{rect}\left(\frac{f}{2B}\right) + \frac{1}{4}e^{-j2\pi f T/4} \qquad \text{ma } X(f) = \sum_n X_n \delta\left(f - \frac{n}{T}\right)$$

con $X_n = \frac{1}{T} X_0\left(f = n/T\right)$.

$$z(t) = \sum_n X_n H(f = n/T)\,\delta(f - n/T)$$

X_n: $n = 1 \to 2/3 + e^{-j\pi/2}$; $n = -1 \to 2/3 + e^{j\pi/2}$; $n = 2 \to -2/3$; $n = -2 \to -2/3$

Il filtro fa passare solo le componenti con $n = \pm 1, \pm 2$

$$Z(f) = -\frac{2}{3}\left\{\delta\left(f - \frac{2}{T}\right) + \delta\left(f + \frac{2}{T}\right)\right\} + \left(\frac{2}{3}e^{j\pi/2} + 1\right)\delta\left(f - \frac{1}{T}\right) + \left(\frac{2}{3}e^{-j\pi/2} + 1\right)\delta\left(f + \frac{1}{T}\right)$$

$$= -\frac{2}{3}\left\{\delta\left(f - \frac{2}{T}\right) + \delta\left(f + \frac{2}{T}\right)\right\} + \frac{2}{3}\left\{\delta\left(f - \frac{1}{T}\right)e^{j\pi/2} + \delta\left(f + \frac{1}{T}\right)e^{-j\pi/2}\right\} + \delta\left(f - \frac{1}{T}\right) + \delta\left(f + \frac{1}{T}\right)$$

da cui, ricavo $z(t) = -\frac{4}{3}\cos\left(2\pi t \frac{2}{T}\right) + \frac{4}{3}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right) + 2\cos\left(\frac{2\pi t}{T}\right)$

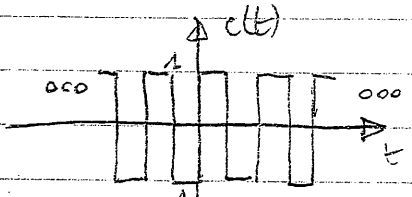
$$P_z = 2|X_1|^2 + 2|X_2|^2 = 2\left(\frac{4}{9} + 1\right) + 2\left(\frac{4}{9}\right) = \frac{16}{9} + 2 = \frac{34}{9}$$

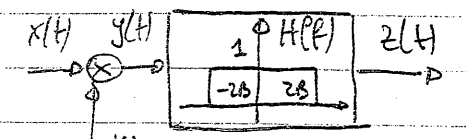
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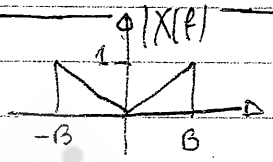
$X(f) = j\left(\frac{f}{B}\right) \text{rect}\left(\frac{f}{2B}\right)$

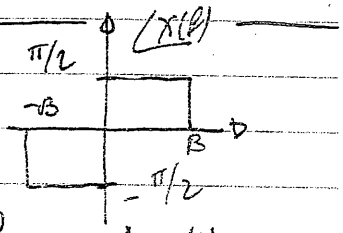




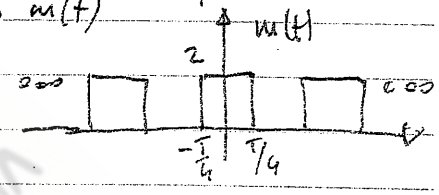
Calcolare: $C(f)$, $z(t)$ con $T = 1/B$

Disegno modulo e fase di $X(f)$:





Per semplificare i conti, utilizzo un segnale d'appoggio $m(t)$



$c(t) = m(t - T/4) - 1 \rightarrow C_k = \frac{1}{T} M_0\left(\frac{k}{T}\right) e^{-j 2\pi \frac{k}{T} \frac{T}{4}} - \delta(k)$

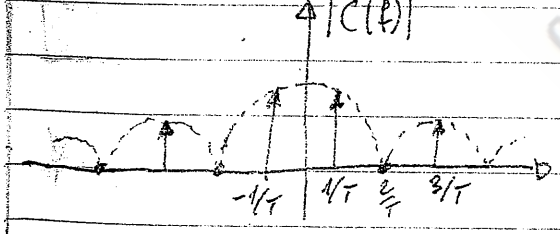
$M_0\left(\frac{k}{T}\right) = 2\frac{T}{2} \text{sinc}\left(\frac{k}{T}\frac{T}{2}\right) = T \text{sinc}\left(\frac{k}{2}\right)$. A questo punto ricavo i C_k

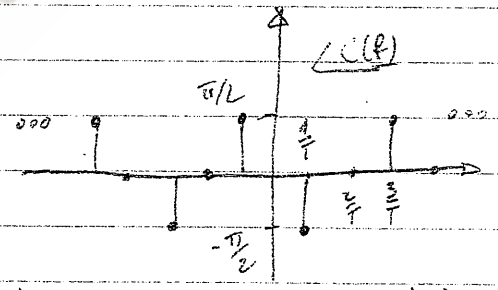
$C_k = \text{sinc}\left(\frac{k}{2}\right) e^{-j\frac{\pi k}{2}} - \delta(k) = \begin{cases} 0 & k \text{ pari} \\ \text{sinc}\left(\frac{k}{2}\right)(-j)^k & k \text{ dispari} \end{cases}$

$C(f) = \sum_k \delta(f - k/T)$

$Y(f) = X(f) \otimes C(f) = X(f) \otimes \sum_k C_k \delta\left(f - \frac{k}{T}\right)$

$= \sum_k C_k X(f - kB) = \sum_{k \text{ disp}} \text{sinc}\left(\frac{k}{2}\right) e^{-j\pi/2 (k-1)} \left(\frac{f - kB}{B} \text{rect}\left(\frac{f - kB}{2B}\right)\right)$





Nel filtro passano solo le componenti $k = \pm 1$, quindi:

$Z(f) = \underbrace{\text{sinc}\left(\frac{1}{2}\right)}_{2/\pi} \left\{ \frac{f - B}{B} \text{rect}\left(\frac{f - B}{2B}\right) - \frac{f + B}{B} \text{rect}\left(\frac{f + B}{2B}\right) \right\}$

$Z(f) = \text{rect}\left(\frac{f}{4B}\right) - 2\text{tri}\left(\frac{f}{4B}\right)$. Antitrasformo $\rightarrow z(t) = 4B\left(\text{sinc}(4Bt) - \text{sinc}^2(2Bt)\right)$